

Superconducting Electroweak Strings

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Classical solutions describing strings endowed with an electric charge and carrying a constant electromagnetic current are constructed within the bosonic sector of the Electroweak Theory. For any given ratio of the Higgs boson mass to W boson mass and for any Weinberg's angle, these strings comprise a family that can be parameterized by values of the current through their cross section, I_3 , by their electric charge per unit string length, I_0 , and by two integers. These parameters determine the electromagnetic and Z fluxes, as well as the angular momentum and momentum densities of the string. For $I_0 \rightarrow 0$ and $I_3 \rightarrow 0$ the solutions reduce to Z strings, or, for solutions with $I_0 = \pm I_3$, to the W-dressed Z strings whose existence was discussed some time ago.

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Superconducting strings have been described by Witten within a $U(1) \times U(1)$ gauge field theory with two complex scalars [1]. One of these scalars is responsible for the vortex formation, while the other one gives rise to a constant current along the string. This phenomenon, dubbed 'bosonic superconductivity', has found numerous applications, mainly in the cosmological context [2], since Witten's model can be viewed as sector of some high energy Grand Unification Theory [1]. One can wonder whether a similar phenomenon could also exist at lower energies, as for example in the Electroweak (EW) Theory. Indeed, one has also in this case a pair of complex scalar fields, while the $U(1) \times U(1)$ group is contained in the $SU(2) \times U(1)$ EW gauge group. However, the only EW strings known up to now have been the Z and W strings (see [3] for a review), but these carry no current.

Recently superconducting (SC) strings have been constructed in the semilocal limit of the EW theory where Weinberg's angle $\theta_w = \pi/2$ and the Yang-Mills field decouples [4]. The aim of the present Letter is to extend this construction to arbitrary values of θ_w . The gauge field of the resulting solutions turns out to be of a quite general non-Abelian and not

of the $U(1) \times U(1)$ type. Below we shall show how to obtain them by deforming Z strings, which gives solutions physically different from Z strings, as is seen by comparing the values of the gauge invariant quantities. A special case of these new solutions are the W-dressed Z strings whose possible existence was discussed some time ago [5, 6].

The EW theory. The bosonic sector of the theory is defined by the action $S = \frac{1}{g_0^2} \int (\mathcal{L}_{\text{YM}} + \mathcal{L}_{\text{H}}) d^4x$ with

$$\begin{aligned} \mathcal{L}_{\text{YM}} &= -\frac{1}{4g^2} W_{\mu\nu}^a W^{a\mu\nu} - \frac{1}{4g'^2} F_{\mu\nu} F^{\mu\nu}, \\ \mathcal{L}_{\text{H}} &= (D_\mu \Phi)^\dagger D^\mu \Phi - \frac{\beta}{8} (\Phi^\dagger \Phi - 1)^2, \end{aligned} \quad (1)$$

where $W_{\mu\nu}^a = \partial_\mu W_\nu^a - \partial_\nu W_\mu^a + \epsilon_{abc} W_\mu^b W_\nu^c$ and $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ are the $SU(2)$ and $U(1)$ field strengths, respectively, and $\Phi^{\text{tr}} = (\phi_+, \phi_-)$ is a doublet of complex Higgs fields with $D_\mu \Phi = (\partial_\mu - \frac{i}{2} A_\mu - \frac{i}{2} \tau^a W_\mu^a) \Phi$, where τ^a are the Pauli matrices. Here all fields and spacetime coordinates have been rendered dimensionless by rescaling, the rescaled gauge couplings are expressed in terms of the Weinberg angle as $g = \cos \theta_w$, $g' = \sin \theta_w$, and the dimensionless masses of the Z, W, and Higgs bosons are given, respectively, by $m_Z = 1/\sqrt{2}$, $m_W = gm_Z$, $m_H = \sqrt{\beta} m_Z$. The mass scale is $g_0 \Phi_0$ where Φ_0 is the dimensionful Higgs field vacuum expectation value and g_0 determines the value of the electric charge, $e = g_0 g g'$. The action is invariant under gauge transformations

$$\Phi \rightarrow U\Phi, \quad \mathbf{W} \rightarrow U\mathbf{W}U^{-1} + 2iU\partial_\mu U^{-1}dx^\mu, \quad (2)$$

where $\mathbf{W} = (A_\mu + \tau^a W_\mu^a)dx^\mu$ and $U \in SU(2) \times U(1)$.

Varying the action gives the field equations,

$$\begin{aligned} \partial_\mu F^{\mu\nu} &= g'^2 \Re(i\Phi^\dagger D^\nu \Phi), \\ \partial_\mu W_a^{\mu\nu} + \epsilon_{abc} W_\sigma^b W^{c\sigma\nu} &= g^2 \Re(i\Phi^\dagger \tau^a D^\nu \Phi), \\ D_\mu D^\mu \Phi &= \frac{\beta}{4} (\Phi^\dagger \Phi - 1)\Phi. \end{aligned} \quad (3)$$

There exist several ways to define the electromagnetic field $\mathcal{A}_{\mu\nu}$ (see e.g. [8]), all of which agree in the Higgs vacuum. In our case it is convenient to adopt the definition using the gauge invariant field tensor [9]

$$\mathcal{W}_{\mu\nu} = \partial_\mu \mathcal{W}_\nu - \partial_\nu \mathcal{W}_\mu - \epsilon_{abc} n^a \partial_\mu n^b \partial_\nu n^c, \quad (4)$$

where $\mathcal{W}_\mu = n^a \mathcal{W}_\mu^a$ and $n^a = (\Phi^\dagger \tau^a \Phi) / (\Phi^\dagger \Phi)$. The gauge invariant \mathcal{A} and \mathcal{Z} fields are then defined as $\mathcal{A}_{\mu\nu} = \frac{g}{g'} F_{\mu\nu} - \frac{g'}{g} \mathcal{W}_{\mu\nu}$ and $\mathcal{Z}_{\mu\nu} = F_{\mu\nu} + \mathcal{W}_{\mu\nu}$. The electromagnetic current density is $J_\mu = \frac{1}{4\pi} \partial^\nu \mathcal{A}_{\nu\mu}$. In the unitary gauge, one can introduce the potentials for the \mathcal{A}, \mathcal{Z} fields: $\mathcal{A}_\mu = \frac{g}{g'} A_\mu - \frac{g'}{g} \mathcal{W}_\mu$ and $\mathcal{Z}_\mu = A_\mu + \mathcal{W}_\mu$.

Symmetry reduction. We shall be considering cylindrically symmetric solutions. Splitting the spacetime coordinates as $x^\mu = (x^\alpha, x^k)$, where $\alpha = 0, 3; k = 1, 2$, and introducing the polar coordinates in the x^k plane, $x^1 + x^2 = \rho e^{i\varphi}$, we assume the system to be invariant under the action of the three spacetime symmetry generators $K = \{\partial/\partial x^\alpha, \partial/\partial \varphi\}$. This implies the conservation of the energy, E , momentum, P , and angular momentum, M , expressed (per unit string length) by $\int T_\mu^0 K^\mu d^2x$ for the three above choices of K , respectively. Here T_ν^μ is the energy-momentum tensor; $d^2x = dx^1 dx^2$.

Let σ_α be a constant (co)vector in the x^α plane. We make the field ansatz

$$\begin{aligned} \mathbf{W} &= u^+(\rho) \sigma_\alpha dx^\alpha + u^-(\rho) d\varphi \\ &+ \sum_{a=1,3} \tau^a [v_a^+(\rho) \sigma_\alpha dx^\alpha + v_a^-(\rho) d\varphi], \quad \phi_\pm = f_\pm(\rho), \end{aligned} \quad (5)$$

where all functions of ρ are real. This ansatz is invariant under complex conjugation and it keeps its form also under Lorentz boosts in the x^α plane. Since the latter preserve the Lorentz norm, $\sigma^2 \equiv -\sigma_\alpha \sigma^\alpha = (\sigma_3)^2 - (\sigma_0)^2$, there can be solutions of three different types: magnetic ($\sigma^2 > 0$), electric ($\sigma^2 < 0$), and chiral ($\sigma^2 = 0$). Introducing $f = f_+ + if_-$ and $v_\pm = v_1^\pm + iv_3^\pm$, also $X_\pm = iu^\pm \bar{f} + f v_\pm$ and $\Lambda = v_+ \bar{v}_- - v_- \bar{v}_+$, the field equations obtained by inserting (5) to Eqs.(3) read

$$\begin{aligned} \hat{\mathcal{D}}_\pm u^\pm &= \frac{g'^2}{2} \Im(f X_\pm), \quad \hat{\mathcal{D}}_\pm v_\pm = \frac{g^2}{2} \bar{f} X_\pm \pm \frac{\Lambda}{2\rho^2} \lambda_\mp v_\mp, \\ \hat{\mathcal{D}}_+ f &= \sum_{\varsigma=+,-} \frac{\lambda_\varsigma}{4} (iu^\varsigma \bar{X}_\varsigma + X_\varsigma \bar{v}_\varsigma) + \frac{\beta}{4} (|f|^2 - 1) f, \\ \Im(\lambda_+ v_+ \bar{v}_+{}' + \lambda_- v_- \bar{v}_-{}' - g^2 f \bar{f}') &= 0, \end{aligned} \quad (6)$$

where $\hat{\mathcal{D}}_\pm = \frac{d^2}{d\rho^2} \pm \frac{d}{d\rho}$ and $\lambda_+ = \sigma^2, \lambda_- = 1/\rho^2$. The first order equation in the last line is compatible with the other equations but constraints the boundary values for their solutions.

Boundary conditions. Assuming that $u^-(0) = k \equiv 2n - \nu$, $v_1^-(0) = 0$, $v_3^-(0) = \nu$ with $n, \nu \in \mathbb{Z}$, the fields (5) can be transformed, via gauge transformation with $U = \exp\{-\frac{i}{2}[\eta + \psi\tau^3]\} \exp\{\frac{i}{2}\gamma\tau^2\}$ where $\psi = V_3^-(0) \varphi - c_1 \sigma_\alpha x^\alpha$, $\eta = k \varphi + c_1 \sigma_\alpha x^\alpha$, c_1 is defined in Eq.(10),

to the gauge where they are regular at the symmetry axis:

$$\begin{aligned}
\mathbf{W} = & (u^+ - c_1) \sigma_\alpha dx^\alpha + (u^- - k) d\varphi \\
& + (\tau^1 \cos \psi + \tau^2 \sin \psi) [V_1^+ \sigma_\alpha dx^\alpha + V_1^- d\varphi] \\
& + \tau^3 [(V_3^+ + c_1) \sigma_\alpha dx^\alpha + (V_3^- - V_3^-(0)) d\varphi] \\
& + \tau^2 d\gamma, \quad \phi_\pm = F_\pm e^{-\frac{i}{2}(\eta \pm \psi)}.
\end{aligned} \tag{7}$$

Here $V_1^\pm + iV_3^\pm = e^{i\gamma} v_\pm$ and $F_+ + iF_- = e^{-\frac{i}{2}\gamma} f$, so that γ determines the mixing between the upper and lower components of the Higgs field. We shall choose either $\gamma = 0$ or $e^{\frac{i}{2}\gamma} = -if/|f|$, referring to these two choices as gauge IIa and gauge IIb, respectively. In both cases $V_3^-(0) \in \mathbb{Z}$, the latter choice corresponding to the unitary gauge, $F_+ = 0$. The regularity of the Higgs field requires also that $F_\pm(0) = 0$ if $V_3^-(0) \pm k \neq 0$. The local solution of Eqs.(6) with such boundary conditions reads

$$\begin{aligned}
u &= a_1 + i(k + a_2 \rho^2) + \dots, \quad v_+ = i + a_4 \rho^\nu + \dots, \\
f &= a_3 \rho^{|n|} + iq \rho^{|n-\nu|} + \dots, \quad v_- = i\nu + ia_5 \rho^2 + \dots
\end{aligned} \tag{8}$$

where a_1, \dots, a_5, q are 6 real integration constants.

At large ρ we require the fields to approach the exact solution of Eqs.(6) (written in the gauge (5))

$$\mathbf{W} = (1 - \tau^3) [(c_1 + Q \ln \rho) \sigma_\alpha dx^\alpha + c_2 d\varphi], \quad \phi_+ = 1, \tag{9}$$

with constant c_1, c_2, Q and with $\phi_- = 0$, which describes fields produced by an electric current distributed along the x^3 axis. The corresponding asymptotic solution of Eqs.(6) reads

$$\begin{aligned}
u &= c_1 + Q \ln \rho + \frac{c_3 g'^2}{\sqrt{\rho}} e^{-m_Z \rho} + i [c_2 + c_4 g'^2 \sqrt{\rho} e^{-m_Z \rho}] + \dots \\
v_+ &= e^{ic_9} \left\{ \frac{c_5}{\sqrt{\rho}} e^{-m_\sigma \rho} - i [c_1 + Q \ln \rho - \frac{c_3 g'^2}{\sqrt{\rho}} e^{-m_Z \rho}] \right\} + \dots \\
v_- &= e^{ic_9} \{ c_6 \sqrt{\rho} e^{-m_\sigma \rho} + i [-c_2 + c_4 g'^2 \sqrt{\rho} e^{-m_Z \rho}] \} + \dots \\
f &= e^{-\frac{i}{2}c_9} \left\{ 1 + \frac{c_7}{\sqrt{\rho}} e^{-m_H \rho} + i \frac{c_8}{\sqrt{\rho}} e^{-m_\sigma \rho} \right\} + \dots
\end{aligned} \tag{10}$$

where the dots stand for the higher order and non-linear terms. This approaches (9) (modulo the phases) exponentially fast, with the rates determined by the masses m_Z, m_H and m_σ ,

where $m_\sigma^2 = m_w^2 + \sigma^2$. We thus have a family of local asymptotic solutions parameterized by the 10 integration constants c_1, \dots, c_9, Q .

To construct the global solutions, we extend the asymptotics (8),(10) and match them in the intermediate region. To fulfill the 16 matching conditions for the 8 functions and for their first derivatives, we have in our disposal 17 free parameters: 6+10 in Eqs.(8),(10) and also σ^2 . There is therefore one extra parameter left after matching, and this will label the global solutions. We choose this parameter to be q in Eq.(8).

The above boundary conditions completely determine the ‘worldsheet current’ $I_\alpha \equiv \int J_\alpha d^2x = -\frac{1}{2gg'} \sigma_\alpha Q$, the \mathcal{Z} flux $\Psi_Z = \int \mathcal{Z}_{\varphi\rho} d^2x = 2\pi(k - V_3^-(0))$, the \mathcal{A} flux $\Psi_A = \int \mathcal{A}_{\varphi\rho} d^2x = \frac{2\pi}{gg'}(V_3^-(0) - c_2) + \frac{g}{g'} \Psi_Z$, and the angular momentum $M = \int T_\varphi^0 d^2x = \frac{2\pi}{(gg')^2} \sigma_0 Q c_2$. Here $V_3^-(0)$ is computed in the unitary gauge IIb where one goes to introduce the potentials to evaluate the fluxes. The energy expresses as $E = \frac{\pi}{(gg')^2}(\sigma_0^2 + \sigma_3^2)Qu^+(\infty) + \epsilon$ and the momentum is $P = \frac{2\pi}{(gg')^2} \sigma_0 \sigma_3 Qu^+(\infty)$, where

$$\epsilon = \int \left(\frac{u_-'^2}{2\rho^2 g'^2} + \frac{|v_-'|^2}{2\rho^2 g^2} + \frac{|X_-|^2}{4\rho^2} + |f'|^2 + \frac{\beta}{8}(|f|^2 - 1)^2 \right) d^2x. \quad (11)$$

Weakly deformed Z strings. If $q = 0$ then setting $f_- = v_1^\pm = 0$, $u^- = 2g'^2(w(\rho) - n) + k$, $v_3^+ = -u^+ = 1$, $v_3^- = 2g^2(w(\rho) - n) + \nu$, Eqs.(6) reduce to

$$\hat{D}_+ f_+ = \left(\frac{w^2}{\rho^2} + \frac{\beta}{4}(f_+^2 - 1) \right) f_+, \quad \hat{D}_- w = \frac{1}{2} f_+^2 w, \quad (12)$$

where $n \leftarrow w \rightarrow 0$ and $0 \leftarrow f_+ \rightarrow 1$ as $0 \leftarrow \rho \rightarrow \infty$. The solutions are the Z strings [3]. In the gauge IIa,

$$\mathbf{W} = 2(g'^2 + g^2 \tau^3)(w - n)d\varphi, \quad \phi_+ = f_+ e^{-n\varphi}, \quad (13)$$

and $\mathcal{Z}_\mu dx^\mu = 2(w - n)d\varphi$, $\mathcal{A}_\mu = 0$, $\Psi_Z = 4\pi n$.

Giving now a small value to q in Eq.(8) produces small deformations of the Z strings. The linear in q deformation terms read (always in the gauge IIa)

$$\delta \mathbf{W} = (\tau^1 \cos \psi + \tau^2 \sin \psi) [\delta v_1^+ \sigma_\alpha dx^\alpha + \delta v_1^- d\varphi], \quad (14)$$

also $\delta \phi_+ = 0$ and $\delta \phi_- = e^{i(\psi - n\varphi)} \delta f_-$. Here δf_- and δv_1^\pm fulfill equations obtained by linearizing Eqs.(6) around the Z string configuration, which can be cast into the eigenvalue problem form

$$\Psi'' = (\sigma^2 + V[\beta, \theta_w, n, \nu, \rho]) \Psi, \quad (15)$$

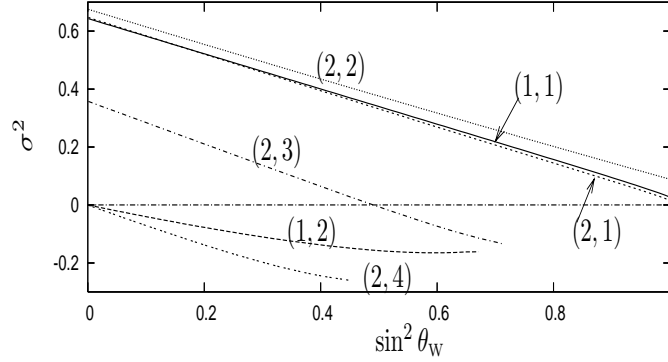


FIG. 1: The eigenvalue $\sigma^2(\beta, \theta_W, n, \nu)$ for the bound state solutions of Eqs.(15) with $\beta = 2$, $n = 1, 2$. In the $\sigma^2 < 0$ region the curves terminate where $m_\sigma = 0$.

supplemented by the linearized version of the constraint equation in (6). Here V is a 3×3 matrix and Ψ is a 3 component vector. Eqs.(15) admit bound state solutions for which $\Psi \sim e^{-m_\sigma \rho}$ as $\rho \rightarrow \infty$. These describe small deformations of Z strings by a current $I_\alpha \sim \sigma_\alpha$, and so the eigenvalue $\sigma^2 = \sigma^2(\beta, \theta_W, n, \nu)$ determines the spacetime type of I_α .

Given β, θ_W and $n \in \mathbb{Z}_+$, Eqs.(15) admit $2n$ different bound state solutions labeled by $\nu = 1, 2, \dots, 2n$. If $\beta > 1$ then n of these solutions have $\sigma^2 > 0$ for $\theta_W \in [0, \frac{\pi}{2}]$. The eigenvalue σ^2 for the other n solutions is not positive definite, and these solutions exist only as long as $m_\sigma^2 = m_W^2 + \sigma^2 > 0$; see Fig.1. Every Z string therefore admits both electric and magnetic current deformations. Chiral deformations are possible if only β, θ_W belong to the curves determined by the condition $\sigma^2(\beta, \theta_W, n, \nu) = 0$; see Fig.2.

Remarkably, these curves coincide with those delimiting the Z string stability regions [10]. In fact, the same Z string perturbations as in Eq.(14) were considered in [7, 10], but choosing $\sigma_\alpha = (\sigma_0, 0)$ instead of $\sigma_\alpha = (\sigma_0, \sigma_3)$. For $\sigma^2 = \sigma_3^2 - \sigma_0^2 < 0$ these two choices are equivalent, since one can always boost away the σ_3 component in this case. However, they are physically different for solution of Eqs.(15) with $\sigma^2 > 0$. Indeed, if $\sigma_3 = 0$ then σ_0 should be imaginary in this case leading to growing in time perturbations, whereas if $\sigma_3 \neq 0$ then σ_α can be a real spacelike vector and the very same solutions will describe the magnetic deformations of Z strings. The regions above/below a $\sigma^2(\beta, \theta_W, n, \nu) = 0$ curve therefore correspond to values of β, θ_W for which the ν -th deformation of the n -th Z string is of the electric/magnetic type, and also to the values for which the ν -th Z string perturbation mode is stable/unstable.

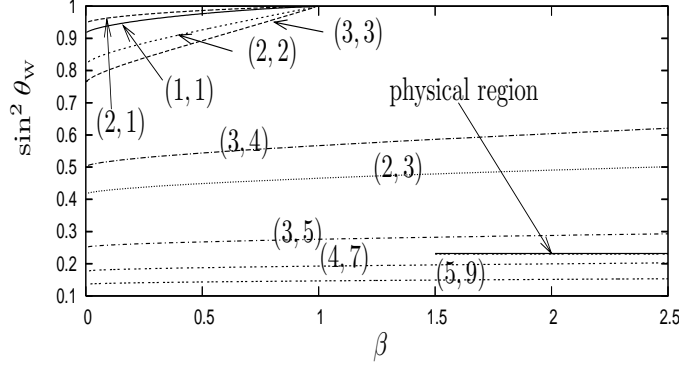


FIG. 2: The $\sigma^2(\beta, \theta_w, n, \nu) = 0$ curves for several values of (n, ν) . Curves with $\nu \leq n$ are confined to the $\beta \leq 1$ region, while those with $\nu > n$ extend to higher values of β .

SC strings. Further increasing the value of q in Eq.(8) promotes the $2n$ weak deformations of a given Z string to $2n$ fully non-linear SC solutions, without however changing the sign of σ^2 , so that the above perturbative picture remains qualitatively valid. The typical solution is shown in Fig.3. These strings carry an electric current breaking their Lorentz boost invariance, such that each solution of Eqs.(6) determines actually a whole family of strings with fixed $\sigma^2 = -\sigma_\alpha \sigma^\alpha$ and with σ_α 's related to each other by boosts. For given $\beta > 0$ and $\theta_w \in [0, \frac{\pi}{2}]$ these solutions can be labeled by values of σ_α , by $n = 1, 2, \dots$ and by $\nu = 1, \dots, 2n$. The related physical parameters of the string are its momentum P , angular momentum M and charge I_0 per unit string length, the \mathcal{A}, \mathcal{Z} magnetic fluxes and the current I_3 through the string cross section. The vector $I_\alpha \sim \sigma_\alpha$ can generically be spacelike or timelike, depending on ν . For magnetic solutions with $I_\alpha I^\alpha < 0$ ($\sigma^2 > 0$) there exists a comoving reference frame where $\sigma_0 = I_0 = P = M = 0$, but their current I_3 never vanishes, which suggests the term ‘superconductivity’. For electric solutions with $I_\alpha I^\alpha > 0$ there exists a reference frame where $\sigma_3 = I_3 = P = 0$, but the charge I_0 never vanishes.

The rest frame values of the charge I_0 , current I_3 , angular momentum M , and also of σ^2 , Ψ_Z , Ψ_A for the $\nu = 1, 2$ SC solutions with $n = 1$, $\beta = 2$, $\sin^2 \theta_w = 0.23$ are shown in Fig.4 as functions of q . We observe that, although for the electric solution the fluxes Ψ_Z , Ψ_A reduce to their Z string values as $q \rightarrow 0$, this does not happen for the magnetic solution, although all SC solutions converge to the same Z string limit as $q \rightarrow 0$ (in the gauge IIa). To understand this we notice that in the unitary gauge IIb used to compute the fluxes one has $V_3^-(0) = -\nu$ for $q = 0$ and $V_3^-(0) = \nu$ for $q \neq 0$ ($\nu < 2n$). SC strings are thus indeed

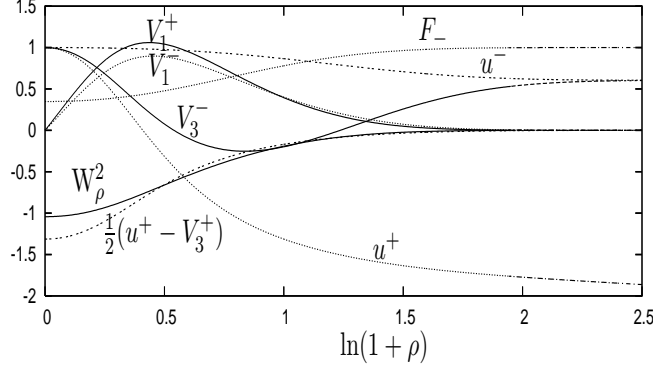


FIG. 3: Magnetic SC solution with $\beta = 2$, $g'^2 = 0.23$, $n = \nu = 1$ and $q = 0.348$ shown in the unitary gauge IIb.

‘topologically different’ and converge to Z strings only pointwise in this gauge. Computing the fluxes gives $\Psi_Z = 4\pi n$ for Z strings, while for the SC strings $\Psi_Z(\nu) = 4\pi(n - \nu)$ for $\nu = 1, \dots, 2n - 1$ and $\Psi_Z(2n) = 4\pi(n - \nu q^2/(q^2 + a_3^2))$.

Since the SC strings are coupled to the electromagnetic field, their energy per unit length (also P) is divergent: E contains the term $Q(\sigma_0^2 + \sigma_3^2)u^+(\infty)$ where $u^+(\rho) \sim \ln \rho$ for $\rho \rightarrow \infty$. This type of divergence arises, however, even for ordinary electrical wires, if they are infinitely long, whereas objects made of finite pieces of strings, such as closed current loops, will have finite energy. E becomes finite if $g = 0$ or $g' = 0$, since the long range massless fields then decouple and $Q = 0$. For $g = 0$ the solutions reduce to the twisted SC semilocal strings studied in [4].

W-dressed Z strings. E is also finite for special chiral solutions with $\sigma_\alpha = I_\alpha = 0$. These can presumably be viewed as superpositions of SC strings with currents flowing in the opposite directions and canceling each other. Chiral solutions are not generic, given $q \neq 0$ they exist only for values of β, θ_w belonging to the curves determined by the condition $\sigma^2(\beta, \theta_w, n, \nu, q) = 0$. For $q \rightarrow 0$ these curves reduce to those shown in Fig.2, while for $q \neq 0$ they remain qualitatively the same but shift *upwards*. As a result, given a point β, θ_w in an (in fact small) upper vicinity of a chosen curve in Fig.2, one can adjust the value of q such that the shifted curve will pass through this point. This fine tuning determines the values of β, θ_w, q giving rise to a solution of Eqs.(6) with $\sigma^2 = 0$. Setting then $\sigma_\alpha = 0$, this solution describes a static, purely magnetic currentless string which is not a gauge copy of a Z string [7] since it has $\Psi_A \neq 0$. The profiles of such solutions are similar to those shown in Fig.3,

but u^+, V_a^+ are no longer needed since they are multiplied by $\sigma_\alpha = 0$ in the ansatz. Such solutions apparently correspond to the W-dressed Z strings discussed some time ago [5, 6]. These were supposed to be strings with a W-condensate at the core, with lower energy than Z strings, and hopefully with better stability. They were looked for numerically [7, 11], but with negative results, presumably because their parameters were not fine tuned. We find that for $n = 1$ these dressed solutions are indeed energetically favored as compared to Z strings with the same β, θ_w . However, the parameters β, θ_w lie then in the unphysical region (see Fig.2). The dressed solutions also exist in the physical region, for $g'^2 = 0.23$ and $1.5 \leq \beta \leq 3.5$, but only for higher values of n, ν starting from $n = 4, \nu = 7$ (see Fig.2), in which case we find them to be more heavy than Z strings.

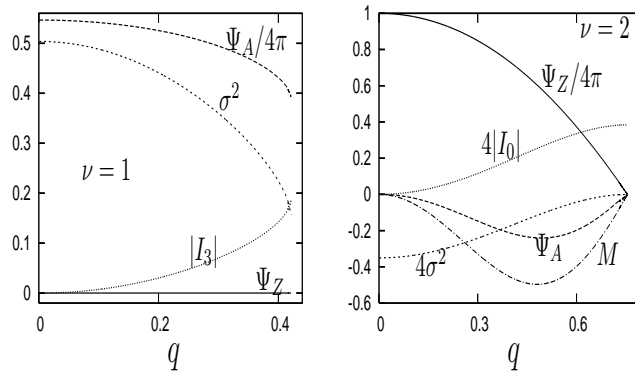


FIG. 4: The parameters of the SC solutions with $n = 1$, $\beta = 2$, $\sin^2 \theta_w = 0.23$, and $\nu = 1, 2$.

Summarizing, we have sketched the construction of new non-topological string solutions in Standard Model. To judge their physical importance their stability should be analyzed.

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